

Spotlights, Floodlights, and the Magic Number Zero: Simple Effects Tests in Moderated Regression

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Web Appendix A

This web appendix is intended to serve as a standalone reference guide for conducting spotlight and floodlight analyses after one has read the main text. These materials repeat and expand upon the two cases discussed in the text and add three additional cases. These building blocks can be combined to examine spotlight or floodlight analyses of any linear model. An analysis template for the base case (Case #0: 2 x Continuous) is given in Table W1.

Case #1: 2 x Continuous when the Manipulation of Z is Within Subjects

Imagine a version of our original example with two levels of the manipulated factor Z (0 = confederate took 2 candies, 1 = confederate took 30 candies,) and a continuous measure of X = BMI. This time, however, let Z be a repeated measures factor. In this case, one simply creates a contrast score for each subject showing the effect of the manipulation for that subject: $Z_{\text{contrast}} = Y_{30} - Y_2$ (see Judd, McClelland, and Ryan 2009 or Keppel and Wickens 2004); one could similarly create contrast scores for within-subject designs with more than 2 levels. One then analyzes the Z_{contrast} scores as a function of X = BMI:

$$(W1) \quad Z_{\text{contrast}} = a + bX$$

Extending the principle of the magic number zero, the test of the intercept, a , in this analysis is the predicted Z_{contrast} score when $X = 0$. The coefficient b now is equivalent to a test of the interaction of X with Z in the original design. To create a spotlight test of the effect of the repeated factor Z at the borderline between normal and overweight, create $X' = X - 25$. Rerun the regression $Z_{\text{contrast}} = a' + b'X'$. Now the test of the intercept a' is the effect of the repeated

factor Z at the new zero point associated with the chosen level of X. An analysis template for Case 1 is given in Table W2.

Case #2: 2 x 2 x Continuous

Often, one may be interested in how a continuous variable moderates a 2 x 2 interaction, resulting in a three-way interaction. For example, in addition to manipulating quantity taken, we might also manipulate the perceived health of the item being considered (candy vs. granola, as in Study 1 of McFerran et al. 2010), with all factors manipulated between-subjects. The prediction might be that attenuation of assimilation only occurs for unhealthy food because participants are cued to be more vigilant when food is unhealthy than when it is healthy. (McFerran et al. found this not to be the case.) The model for this design is:

$$(W2) \quad Y = a + bZ + cW + dX + eZW + fZX + gWX + hZWX$$

Z and X are coded the same as they were in the opening example, and W is coded 0 for candy and 1 for granola. If the parameter *h* testing the three-way interaction is significant, it becomes relevant to test the simple interaction of two of the variables at different levels of the third variable. The coefficient *e* tests the simple ZW interaction when X = 0. (It does *not* test the ZW interaction that would be evident in plotting the ZW cell means, collapsing over levels of X.) The coefficient *f* tests the simple ZX interaction when W = 0. The coefficient *g* tests the simple WX interaction when Z = 0. To follow up a simple two-way interaction, one tests the simple-simple effect of one of the variables holding constant the other two. In this model, *b* represents the simple-simple effect of Z when W = 0 and X = 0; *c* represents the simple-simple effect of W when X = 0 and Z = 0; and *d* represents the simple-simple effect of X when Z = 0 and W = 0.

Zero is a magic number in this analysis too. Every coefficient is interpreted as the effect of that variable (or interaction) when all variables with which that term interacts are set to 0 because they drop out of the model. Suppose that we obtained a significant three-way interaction ZWX and wanted to follow up with tests of the simple ZW interaction at meaningful levels of X. We would recode $X' = X - 25$ just as in each of the previous examples. When $X' = 0$, the new coefficient on ZW, e' , represents the simple interaction between quantity taken and perceived healthiness when $X' = 0$ which corresponds to a BMI of 25.

Spotlight analysis in a 2 x 2 x Continuous design requires application of the exact same principle used in the previous cases: recoding variables such that 0 represents the value of a variable at which you are interested in the simple effect of the other variables. An analysis plan for Case 2 is given in Table W3.

If one is interested in testing a 2 x 2 x Continuous design where the second factor, W, is manipulated within-subject, some effects are within-subject effects and some effects are between-subject effects. To test these, one can combine the strategies from Case #1 and Case #2. To examine the between-subject effects (“main effects” of Z, X, and the ZX interaction), calculate $W_{Average}$ as in W3 and regress it on Z, X, and ZX as in W4.

$$(W3) \quad W_{Average} = (Y_{W=1} + Y_{W=0}) / 2$$

$$(W4) \quad W_{Average} = a + bZ + cX + dZX$$

a represents the estimate of Y where $Z = 0$ and $X = 0$, averaged across levels of W. b represents the simple effect of Z on Y at $X = 0$, averaged across levels of W. c represents the simple effect of X on Y at $Z = 0$, averaged across levels of W. d represents how the effect of Z on Y changes with X, averaged across levels of W. One could similarly calculate $Z_{Average}$ in Case

#1, though it is unlikely to be a useful metric if one is interested in the within-subject manipulation.

To examine the within-subject effects (all terms involving W), calculate W_{Contrast} as in W5 and regress it on Z, X, and ZX as in W6.

$$(W5) \quad W_{\text{Contrast}} = Y_{W=1} - Y_{W=0}$$

$$(W6) \quad W_{\text{Contrast}} = a' + b'Z + c'X + d'ZX$$

a' represents the simple simple effect of W at $Z = 0$ and $X = 0$. b' represents the simple interaction of Z with W on Y at $X = 0$. c' represents the simple interaction of X with W on Y at $Z = 0$. d' represents the three-way interaction of Z, X, and W on Y.

Case #3: 3 x Continuous

Imagine that we are replicating the original 2 x Continuous example with a hypothetical variant of McFerran et al. (2010). Instead of manipulating Z at two levels, we add a third condition in which the confederate stands next to the candy but never has an opportunity to take any candy. Now there are three between-subjects conditions: take a large quantity; take a small quantity; no opportunity to take any quantity. Will participants be affected by the mere presence of another when that confederate does not have an opportunity to make a choice? This requires a slightly different analysis plan, but uses the same basic principle of the magic number zero. Of course, one needs $k-1$ dummy variables (or contrast coded variables) to represent k levels of a categorical variable. With a 3-level variable, two dummy variables are required. For our example, Z_1 is coded 1 for small quantity, 0 for large quantity or no opportunity, and Z_2 is coded 1 for large quantity, 0 for small quantity or no opportunity. We also need two variables to represent the 2 degrees of freedom interactions, Z_1X and Z_2X . The base model for this design is:

$$(W7) \quad Y = a + b_1Z_1 + b_2Z_2 + cX + d_1Z_1X + d_2Z_2X$$

This equation can be rewritten as:

$$(W7a) Y = (a + cX) + (b_1 + d_1X)Z_1 + (b_2 + d_2X)Z_2$$

Written this way, it becomes clear that the effect of Z_1 is $(b_1 + d_1X)$, so b_1 represents the effect of Z_1 (i.e., the difference between the small quantity and no opportunity conditions) when X is equal to 0; similarly, the effect of Z_2 is $(b_2 + d_2X)$, so b_2 represents the effect of Z_2 (i.e., the difference between the large quantity and no opportunity conditions) when X is equal to 0.

Interpreting X also requires great care. The equation can be rewritten again as:

$$(W7b) Y = (a + b_1Z_1 + b_2Z_2) + (c + d_1Z_1 + d_2Z_2)X$$

Writing it in this form makes it clear that the effect of X is $(c + d_1Z_1 + d_2Z_2)$, so c represents the effect of X when Z_1 and Z_2 are both equal to 0. In the current example, this means that c represents the simple slope of X for the no opportunity condition. Table W4A presents the statistical analysis of illustrative fictitious data ($N = 150$; $M_{\text{BMI}} = 21.81$, $SD_{\text{BMI}} = 2.91$). These are the same data for the small and large quantity groups presented in the base case, with a third group of observations added for the condition in which there is no opportunity for the confederate to take any candy. With this coding, the slope for X (BMI) pertains only to the no opportunity group, revealing a flat line ($c = 0.02$, $t(144) = 0.12$, $p = .90$).

To understand the effect of small quantity vs. no opportunity or large quantity vs. no opportunity, it is important to recode X —otherwise b_1 and b_2 represent the effects for a confederate with a BMI of 0. Setting $X' = X - 25$ and examining the coefficients on Z_1 and Z_2 , we can consider the effects of confederates who are borderline overweight. Table W4B presents the results after the recoding of X . Estimated for confederates with BMI = 25 (i.e., $X' = 0$), taking a larger quantity of candies significantly increased the number of candies taken by the observer ($b_2' = 3.12$, $t(144) = 2.98$, $p = .003$) relative to the no opportunity condition, whereas

taking a smaller quantity of candies did not significantly decrease the number of candies taken by the observer relative to the no opportunity condition ($b_1' = -1.27$, $t(144) = -1.30$, $p = 0.19$).

In comparing Tables W4A and W4B, note that the recoding of X did not affect either the estimates or the tests of the slope for X or either of the products involving X. The last three rows of the two tables are identical. The only change from recoding X is that now the differences between the three groups are measured at BMI = 25 instead of the substantively meaningless BMI = 0.

Also note that although the data for the large and small quantity groups are the same as before, the product terms $X'Z_1$ and $X'Z_2$ do not show significant interactions. Even though the slopes for large and small quantity groups differ significantly, that is not tested by the present coding. Instead, the slope of each group is tested against the slope for the no opportunity group, which has a slope near zero between the other two slopes. If we want to analyze the difference between the small quantity and large quantity conditions, we would recode Z_2 such that it is coded 1 for no opportunity and 0 for small quantity or large quantity. The coefficient on Z_1 would then represent the difference between the small quantity and large quantity conditions. This would reveal a significant interaction. An analysis plan for Case 3 is given in Table W5.

Case #4: Continuous x Continuous

Spotlight analysis is typically used when the model includes the product of a typically manipulated, typically dichotomous factor and a typically measured, continuous factor. It may just as easily be used whether each factor is manipulated or measured, and when both factors are continuous. Consider a field study extension of the basic design that does not rely on the use of confederates: quantities taken by the observed consumer (no longer confederates) are now continuous and measured rather than dichotomous and manipulated. BMI's of the observed

consumer are still continuous and measured. This change in the design clearly has implications for internal validity since observed consumers are no longer randomly assigned to quantity conditions, but we are focused on the statistical analysis. This model is exactly the same as in our original example of the 2 x Continuous variant of McFerran et al. (2010). The model is:

$$(W8) \quad Y = a + bZ + cX + dZX$$

Z represents quantity taken, but now it represents a continuous variable, not a dichotomous one. The procedure remains the same and the interpretation of a , c , and d is analogous to before. To interpret the effect of (continuously varying) quantity on quantity taken by the participant, we again shine the spotlight on borderline overweight individuals by setting $X' = X - 25$ and re-estimating the model $Y = a' + b'Z + c'X' + d'ZX'$. Here, b' represents the effect of a one unit change in quantity taken by the observed consumer on quantity taken by the participant, holding constant the observed consumer's BMI at borderline overweight. (There is no analysis template for Case #4 or Case #5 because the same principles from Table W1 apply.)

Case #5: Quadratic

An important but not immediately obvious application of the Continuous x Continuous case is that it can aid in the interpretation when the simple slope in a quadratic model is significant. A quadratic model is essentially a variable interacting with itself. Imagine that we are examining the effect of quantity taken by thin confederates. We might find that assimilation is attenuated at very high levels of quantity taken, and so we are interested in how the effect of the marginal piece of candy taken varies as a function of how many candies are taken by the confederate. Our model would be:

$$(W9) \quad Y = a + bZ + cZ^2.$$

The simple effect of Z (quantity taken as a continuous variable) is its derivative: $b + 2cZ$. Once again, the coefficient b represents the simple effect when Z is equal to 0. If we are interested in the effect of taking one more candy when the confederate has already taken 10 candies, we can calculate $Z' = Z - 10$ and regress Y on Z' and Z'^2 . The coefficient on Z' represents the marginal effect when $Z' = 0$ or, equivalently, when $Z = 10$. One can interpret this simple effect as testing the slope of the line tangent to the curve for $Z = 10$.

BASE CASE 0: SIMPLE EFFECTS IN A 2 x CONTINUOUS DESIGN

$$Y = a + bZ + cX + dZX$$

A. BASELINE ANALYSIS

	Intercept	Manipulation	Measured Variable	Manipulation * Measured
		Z	X	ZX
Coding		0 = Control 1 = Treatment	Raw scale	
Coefficient	a	b	c	d
Interpretation	Estimate of Y when Z = 0 and X = 0, i.e., for Control group when X = 0	Simple effect of Treatment vs. Control when X = 0	Simple slope of Measured Variable on Y when Z = 0, i.e., for Control group	Change in effect of Treatment vs. Control when Measured Variable increases by 1 unit

B. TEST THE SIMPLE EFFECT OF TREATMENT VS. CONTROL AT FOCAL VALUE $X = X_{Focal}$ BY RECODING X SO THAT IT DROPS OUT OF THE EQUATION

		Z	X'	ZX'
Coding		0 = Control 1 = Treatment	$X' = X - X_{Focal}$	
Coefficient	a'	b'	c'	d'
Equivalent to	$a + cX_{Focal}$	$b + dX_{Focal}$	c	d
Interpretation	Estimate of Y when Z = 0 and X' = 0, i.e., for Control group when X = X_{Focal}	Simple effect of Treatment vs. Control when X' = 0, i.e., when X = X_{Focal}	Simple slope of Measured Variable on Y when Z = 0, i.e., for Control group	Change in effect of Treatment vs. Control when Measured Variable increases by 1 unit

C. TEST THE SIMPLE SLOPE OF X IN TREATMENT GROUP BY RECODING Z SO THAT IT DROPS OUT OF THE EQUATION

		Z''	X	Z''X
Coding		1 = Control 0 = Treatment	Raw scale	
Coefficient	a''	b''	c''	d''
Equivalent to	$a + b$	$-b$	$c + d$	$-d$
Interpretation	Estimate of Y when Z'' = 0 and X = 0, i.e., for Treatment group when X = 0	Simple effect of Treatment vs. Control when X = 0	Simple slope of Measured Variable on Y when Z'' = 0, i.e., for Treatment group	Difference in slope of Measured Variable between Control (Z'' = 1) and Treatment (Z'' = 0)

D. THE CONSEQUENCES OF USING CONTRAST CODES RATHER THAN DUMMY CODES

		Z'''	X	Z'''X
Coding		-1 = Control 1 = Treatment	Raw scale	
Coefficient	a'''	b'''	c'''	d'''
Equivalent to	$a + b/2$	$b/2$	$c + d/2$	$d/2$
Interpretation	Estimate of Y when Z''' = 0 and X = 0, i.e., unweighted average of group estimates when X = 0	Half of simple effect of Treatment vs. Control when X = 0	Simple slope of Measured Variable on Y when Z''' = 0, i.e., unweighted average of group slopes	Half of difference in slope of Measured Variable between Control (Z''' = -1) and Treatment (Z''' = 1)

Table W1. Simple effects in a 2 x Continuous design.

CASE 1: SIMPLE EFFECTS IN A 2 (WITHIN) x CONTINUOUS DESIGN

$$Y_2 - Y_1 = a + bX$$

A. BASELINE ANALYSIS

	Intercept	Measured Variable
		X
Coding		Raw scale
Coefficient	a	b
Interpretation	Estimate of $Y_2 - Y_1$ when $X = 0$, i.e., Simple effect of Treatment vs. Control when $X = 0$	Slope of Measured Variable on ($Y_2 - Y_1$), i.e., Change in effect of Treatment vs. Control when Measured Variable increases by 1 unit

B. TEST THE SIMPLE EFFECT OF MANIPULATION AT FOCAL VALUE $X = X_{\text{Focal}}$ BY RECODING X SO THAT IT DROPS OUT OF THE EQUATION

		X'
Coding		$X' = X - X_{\text{Focal}}$
Coefficient	a'	b'
Equivalent to	$a + bX_{\text{Focal}}$	b
Interpretation	Estimate of $Y_2 - Y_1$ when $X' = 0$, i.e., Simple effect of Treatment vs. Control when $X = X_{\text{Focal}}$	Slope of Measured Variable on ($Y_2 - Y_1$), i.e., Change in effect of Treatment vs. Control when Measured Variable increases by 1 unit

Table W2. Simple effects in a 2 (Within) x Continuous design.

CASE 2: SIMPLE EFFECTS IN A 2 x 2 x CONTINUOUS DESIGN

$$Y = a + bZ + cW + dX + eZW + fZX + gWX + hZWX$$

A. BASELINE ANALYSIS

	Intercept	Manipulation 1	Manipulation 2	Measured Variable	Manipulation 1 * Manipulation 2	Manipulation 1 * Measured Variable	Manipulation 2 * Measured Variable	Manipulation 1 * Manipulation 2 * Measured Variable
		Z	W	X	ZW	ZX	WX	ZWX
Coding		0 = Control 1 = Treatment	0 = Group A 1 = Group B	Raw scale				
Coefficient	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
Interpretation	Estimate of Y when Z = 0, W = 0, X = 0, i.e., for Control Group A when X = 0	Simple simple effect of Treatment vs. Control when W = 0, X = 0, i.e., simple simple effect of Manipulation 1 for Group A when X = 0	Simple simple effect of Group B vs. Group A when Z = 0, X = 0, i.e., simple simple effect of Manipulation 2 for the Control group when X = 0	Simple simple slope of Measured Variable on Y when Z = 0, W = 0, i.e., for Control Group A	Simple interaction of Manipulation 1 x Manipulation 2 when X = 0	Change in effect of Treatment vs. Control when Measured Variable increases by 1 unit for W = 0, i.e., for Group A.	Change in effect of Group B vs. Group A when Measured Variable increases by 1 unit for Z = 0, i.e., for the Control group.	Change in Manipulation 1 x Manipulation 2 interaction when Measured Variable increases by 1 unit

B. TEST THE SIMPLE INTERACTION OF MANIPULATION 1 (Z) x MANIPULATION 2 (W) AT FOCAL VALUE X = X_{Focal} BY RECODING X SO THAT IT DROPS OUT OF THE EQUATION

		Z	W	X'	ZW	ZX'	WX'	ZWX'
Coding		0 = Control 1 = Treatment	0 = Group A 1 = Group B	$X' = X - X_{Focal}$				
Coefficient	<i>a'</i>	<i>b'</i>	<i>c'</i>	<i>d'</i>	<i>e'</i>	<i>f'</i>	<i>g'</i>	<i>h'</i>
Equivalent to	$a + dX_{Focal}$	$b + fX_{Focal}$	$c + gX_{Focal}$	<i>d</i>	$e + hX_{Focal}$	<i>f</i>	<i>g</i>	<i>h</i>
Interpretation	Estimate of Y when Z = 0, W = 0, X' = 0, i.e., for Control Group A when X = X _{Focal}	Simple simple effect of Treatment vs. Control when W = 0, X' = 0, i.e., simple simple effect of Manipulation 1 for Group A when X = X _{Focal}	Simple simple effect of Group B vs. Group A when Z = 0, X' = 0, i.e., simple simple effect of Manipulation 2 for the Control group when X = X _{Focal}	Simple simple slope of Measured Variable on Y when Z = 0, W = 0, i.e., for Control Group A	Simple interaction of Manipulation 1 x Manipulation 2 when X' = 0, i.e., when X = X _{Focal}	Change in effect of Treatment vs. Control when Measured Variable increases by 1 unit for W = 0, i.e., for Group A.	Change in effect of Group B vs. Group A when Measured Variable increases by 1 unit for Z = 0, i.e., for the Control group.	Change in Manipulation 1 x Manipulation 2 interaction when Measured Variable increases by 1 unit

C. TEST THE SIMPLE INTERACTION OF MANIPULATION 1 (Z) BY X AT LEVEL W = W_{New0} BY RECODING W SO THAT IT IS 0 FOR THAT LEVEL SO THAT W DROPS OUT OF THE EQUATION

		Z	W''	X	ZW''	ZX	W''X	ZW''X
Coding		0 = Control 1 = Treatment	1 = Group A 0 = Group B	Raw scale				
Coefficient	a''	b''	c''	d''	e''	f''	g''	h''
Equivalent to	$a + c$	$b + e$	$-c$	$d + g$	$-e$	$f + h$	$-g$	$-h$
Interpretation	Estimate of Y when Z = 0, W'' = 0, X = 0, i.e., for Control Group B when X = 0	Simple simple effect of Treatment vs. Control when W'' = 0, X = 0, i.e., simple effect of Manipulation 1 for Group B when X = 0	Simple simple effect of Group A vs. Group B when Z = 0, X = 0, i.e., simple effect of Manipulation 2 for the Control group when X = 0	Simple simple slope of Measured Variable on Y when Z = 0, W'' = 0, i.e., for Control Group B	Simple interaction of Manipulation 1 x Manipulation 2 when X = 0	Difference in slope of Measured Variable between Treatment (Z = 1) and Control (Z = 0) when W'' = 0, i.e., for Group B.	Difference in slope of Measured Variable between Group A (W'' = 1) and Group B (W'' = 0) when Z = 0, i.e., for the Control group.	Difference in slope of Measured Variable between Group A vs. Group B difference in slope of Measured Variable between Treatment and Control

Note—In conducting an ANOVA of three dichotomous variables, one will typically report main effects, two-way interactions, and the three-way interaction. In the analysis above, we emphasize simple-simple effects and simple interactions. In the 2 x 2 x continuous case, one may easily elicit the analogous overall effect terms, rather than simple effect terms, through judicious recoding such that 0 represents the average rather than one condition or a focal point. These are of particular interest if higher-order interactions are not statistically significantly different from 0.

- To examine the effect of one variable averaged across the sample (analogous to a main effect in ANOVA), mean-center all interacting variables. For example, given equal cell sizes, to examine the effect of X averaged across the sample, code Z such that -1 = Control, 1 = Treatment and code W such that -1 = Group A, 1 = Group B. Note that each coefficient reflects a one-unit change, so the difference between two contrast-coded conditions (2 units) is twice the coefficient (as shown in Table W1D).
- To examine the effect of a two-way interaction averaged across the sample (analogous to a two-way interaction in ANOVA), mean-center the interacting variable. For example, to examine the interaction effect of X and W averaged across the sample, mean-center Z, which in the equal N case, leads to contrast codes -1 = Control, 1 = Treatment.
- To examine the interaction effect of Z and W, mean-center X.
- None of these recoding schemes impacts the implications of the three-way interaction term, though its estimate may nominally change if one scale has been expanded (e.g., 0, 1 to -1, 1) or reversed (e.g., 0, 1 to 1, 0).

Table W3. Simple effects in a 2 x 2 x Continuous design.

A.

Variable	Coefficient	Estimate	Standard Error	<i>t</i>	<i>p</i>
intercept	<i>a</i>	9.21	3.51	2.63	0.01
small quantity taken	<i>b</i> ₁	-5.56	5.07	-1.10	0.27
large quantity taken	<i>b</i> ₂	13.00	5.05	2.57	0.01
confederate BMI	<i>c</i>	0.02	0.16	0.12	0.90
small quantity taken * confederate BMI	<i>d</i> ₁	0.17	0.23	0.75	0.45
large quantity taken * confederate BMI	<i>d</i> ₂	-0.40	0.23	-1.70	0.09

B.

Variable	Coefficient	Estimate	Standard Error	<i>t</i>	<i>p</i>
intercept	<i>a</i> '	9.71	0.74	13.20	0.0001
small quantity taken	<i>b</i> ₁ '	-1.27	0.97	-1.30	0.19
large quantity taken	<i>b</i> ₂ '	3.12	1.05	2.98	0.003
confederate BMI – 25	<i>c</i> '	0.02	0.16	0.12	0.90
small quantity taken * (confederate BMI – 25)	<i>d</i> ₁ '	0.17	0.23	0.75	0.45
large quantity taken * (confederate BMI – 25)	<i>d</i> ₂ '	-0.40	0.23	-1.70	0.09

Table W4. Regression results of fictitious illustrative data for the 3 x Continuous case, where the confederate takes either 2 candies ($Z_1 = 1, Z_2 = 0$), 30 Candies ($Z_1 = 0, Z_2 = 1$), or the confederate does not have an opportunity to take any candies ($Z_1 = 0, Z_2 = 0$). Panel A: X is the confederate's BMI. Panel B: X' is the confederate's BMI – 25.

CASE 3: SIMPLE EFFECTS IN A 3 x CONTINUOUS DESIGN

$$Y = a + b_1Z_1 + b_2Z_2 + cX + d_1Z_1X + d_2Z_2X$$

A. BASELINE ANALYSIS

	Intercept	Manipulation	Manipulation	Measured Variable	Manipulation * Measured Variable	Manipulation * Measured Variable
		Z_1	Z_2	X	Z_1X	Z_2X
Coding		0 = Control 1 = Treatment 1 0 = Treatment 2	0 = Control 0 = Treatment 1 1 = Treatment 2	Raw scale		
Coefficient	a	b_1	b_2	c	d_1	d_2
Interpretation	Estimate of Y when $Z_1 = 0$, $Z_2 = 0$, and $X = 0$, i.e., for Control group when $X = 0$	Simple effect of Treatment 1 vs. Control when $X = 0$	Simple effect of Treatment 2 vs. Control when $X = 0$	Simple slope of Measured Variable on Y when $Z_1 = 0$ and $Z_2 = 0$, i.e., for Control group	Change in effect of Treatment 1 vs. Control when Measured Variable increases by 1 unit	Change in effect of Treatment 2 vs. Control when Measured Variable increases by 1 unit

*B. TEST THE SIMPLE EFFECT OF TREATMENT 1 VS. CONTROL AT FOCAL VALUE $X = X_{Focal}$
BY RECODING X SO THAT IT DROPS OUT OF THE EQUATION*

		Z_1	Z_2	X'	Z_1X'	Z_2X'
Coding		0 = Control 1 = Treatment 1 0 = Treatment 2	0 = Control 0 = Treatment 1 1 = Treatment 2	$X' = X - X_{Focal}$		
Coefficient	a'	b_1'	b_2'	c'	d_1'	d_2'
Equivalent to	$a + cX_{Focal}$	$b_1 + d_1X_{Focal}$	$b_2 + d_2X_{Focal}$	c	d_1	d_2
Interpretation	Estimate of Y when $Z_1 = 0$, $Z_2 = 0$, and $X' = 0$, i.e., for Control group when $X = X_{Focal}$	Simple effect of Treatment 1 vs. Control when $X' = 0$, i.e., when $X = X_{Focal}$	Simple effect of Treatment 2 vs. Control when $X' = 0$, i.e., when $X = X_{Focal}$	Simple slope of Measured Variable on Y when $Z_1 = 0$ and $Z_2 = 0$, i.e., for Control group	Change in effect of Treatment 1 vs. Control when Measured Variable increases by 1 unit	Change in effect of Treatment 2 vs. Control when Measured Variable increases by 1 unit

C. TEST THE SIMPLE SLOPE OF X IN TREATMENT 1 BY RECODING Z_1 SO THAT IT DROPS OUT OF THE EQUATION

		Z_1''	Z_2	X	$Z_1''X$	Z_2X
Coding		1 = Control 0 = Treatment 1 0 = Treatment 2	0 = Control 0 = Treatment 1 1 = Treatment 2	Raw scale		
Coefficient	a''	b_1''	b_2''	c''	d_1''	d_2''
Equivalent to	$a + b_1$	$-b_1$	$b_2 - b_1$	$c + d_1$	$-d_1$	$d_2 - d_1$
Interpretation	Estimate of Y when $Z_1'' = 0$, $Z_2 = 0$, and $X = 0$, i.e., for Treatment 1 group when $X = 0$	Simple effect of Control vs. Treatment 1 when $X = 0$	Simple effect of Treatment 2 vs. Treatment 1 when $X = 0$	Simple slope of Measured Variable on Y when $Z_1'' = 0$ and $Z_2 = 0$, i.e., for Treatment 1 group	Difference in slope of Measured Variable between Control ($Z_1'' = 1$) and Treatment 1 ($Z_1'' = 0$)	Difference in slope of Measured Variable between Treatment 2 ($Z_2 = 1$) and Treatment 1 ($Z_2 = 0$)

Table W5. Simple effects in a 3 x Continuous design.

Web Appendix B

Why Not Test at Plus and Minus One Standard Deviation?

Researchers most commonly will first mean-center their continuous measure, and then transform it to spotlight the effect of the manipulation at plus and minus one standard deviation from the mean; these values were suggested by Cohen and Cohen (1983) for cases where there are no substantively meaningful values. This approach is not wrong, but we would argue that it is suboptimal and we cannot generate a case where it would be the preferred approach. Using the BMI example, depending on the sample, one standard deviation above the mean might be “normal” weight or it might be clinically obese. Moreover, it is hard to argue that we should be more interested in the effect of Z at exactly one standard deviation above the mean of X in this particular sample than in values slightly higher or slightly lower.

There are three main problems of testing at plus and minus one standard deviation. First, if the distribution of the moderator X is skewed, one of those values can be outside the range of the data. Second, if the moderator X is on a coarse scale, it may be impossible to have a value of X exactly equal to plus or minus one standard deviation. Third, if two researchers replicate the same study with samples of very different mean levels of the moderator, it can appear that they fail to replicate each other even when they find exactly the same regression equation in raw score units. This problem is exacerbated by the tendency for authors using the plus and minus one standard deviation approach to fail to report the mean and standard deviation of X.

As an example of these issues and how to solve them, Fernbach et al. (2013) exposed respondents to product concepts that gave high, medium, or low levels of causal detail for why and how a new product delivered a claimed benefit and asked how well respondents understood the concept. Fernbach et al. showed that people who score high on Frederick’s (2005) Cognitive

Reflection Test (CRT) believed that they understood the concept better when more causal detail was given, but people who scored low believed they understood the concept better when low causal detail was given. The CRT scale is a 3-item quiz resulting in a score of 0, 1, 2, or 3 questions right. Fernbach et al. chose to report the test of the simple effect of the manipulation of causal detail at real possible scores of 0 and 3 rather than to report tests at fractional values of CRT at plus and minus one standard deviation from the mean. Fractional scores are impossible for any individual participant to receive.

Moreover, in this particular example the distribution of CRT is skewed, so it would be easy to have a value one standard deviation below the mean that is actually below zero in a particular sample. It would have been meaningless to test such a simple effect that is outside the range of the data. For example, Frederick (2005) reports distributions of CRT scores across several different institutions. Had Fernbach et al. (2013) tested their hypothesis one standard deviation above and below the mean with Frederick's MIT sample ($M = 2.18$, $SD = 0.94$), one standard deviation above the mean would have been an impossibly high value. If they tested their hypothesis one standard deviation above and below the mean with Frederick's University of Toledo sample ($M = 0.57$, $SD = 0.87$), one standard deviation below the mean would have been an impossibly low value. Unfortunately, the modal reporting strategy is to omit reporting the distribution of the individual difference measure in the sample. Had Fernbach conducted one study at MIT and one at Toledo and examined simple effects at one standard deviation above and below the means without reporting the sample distribution, the simple effects would have appeared to be inconsistent because the distributions of CRT differed so much. Determining whether results replicated across samples without knowing their distributions would be impossible. This is not a problem for interpreting a study in isolation, but for comparing studies.